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ABSTRACT (Maximum 200 words)

We addressed the problem of detecting targets using an array of active sensors. We have been concerned with devising means of obtaining reliable detection with a small number of samples (small relative to the number of unknown parameters). This problem arises with large arrays, and/or low cross section targets. Past techniques for addressing this problem incorporated prior structure into likelihood procedures. Such approaches are (i) intractable, requiring iterative solution, (ii) not CFAR, and (iii) not optimal. We have approached this problem using group symmetries.

Specifically, we introduce a framework for exploring array detection problems in a reduced dimensional space by exploiting the theory of invariance in hypothesis testing. This involves calculating a low dimensional basis set of functions called the maximal invariant, the statistics of which are often tractable to obtain, thereby making analysis feasible and facilitating the search for tests with some optimality property. Using this approach, we obtain a locally most powerful test for the unstructured covariance case and show that the Kelly and AMF detectors form an algebraic span for any invariant detector. Applying the same framework to structured covariance matrices, we gain some insights and propose several new detectors which are shown to outperform existing detectors.

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Final Report on "Enhanced Convergence Adaptive Detection"

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Abstract

In our work we introduce a framework for exploring array detection problems in a reduced dimensional space by exploiting the theory of invariance in hypothesis testing. This involves calculating a low dimensional basis set of functions called the maximal invariant, the statistics of which are often tractable to obtain, thereby making analysis feasible and facilitating the search for tests with some optimality property. Using this approach, we obtain a locally most powerful test for the unstructured covariance case and show that the Kelly and AMF detectors form an algebraic span for any invariant detector. Applying the same framework to structured covariance matrices, we gain some insights and propose several new detectors which are shown to outperform existing detectors.

In this final report we explain the key components of this work. The most detailed compilation of our work to date is found in a recently submitted paper [7], a copy of which we have enclosed.

Introduction

The problem of detecting a signal vector of known direction but unknown strength in Gaussian noise whose covariance matrix is unknown has received much attention lately. In [6], Reed et al used the sample covariance estimate from secondary (signal free) data vectors to derive a weight vector for use in an adaptive matched filter (AMF) detector. Kelly[2] used the Generalized Likelihood Ratio (GLR) procedure to derive a constant false alarm rate (CFAR) test. Both methods assume that the covariance matrix is completely unknown (unstructured). In many applications, however, the array geometry and partial information of the noise environment (number of interferers, rough bearing estimates etc.) impose a structure on the covariance matrix. It has been shown in [1] and [3] that the use of structured covariance estimates results in a significant improvement in performance in terms of gain in PD and reduction in the number of secondary data vectors required.

In this research, we introduce a framework for studying the optimality properties of these tests. We consider the following structure for the covariance matrix:

$$R = \Psi B \Psi^H + \lambda R_0 \tag{1}$$

where $R(N \times N)$ is the covariance matrix, $\Psi(N \times d)$ spans a rank-d subspace and R_0 is a known covariance matrix. For this research, we assume that Ψ is known while B and λ are not. This structure not only corresponds to the case of a low rank interference component in a dominant subspace (which frequently arises in narrow-band processing when the noise has an interference component due to a small number of sources superimposed on the receiver noise which is usually white); but also as a special case reduces to the unstructured matrix when d equals N. We shall therefore work with this model to obtain general results which can then be applied to specific instances.

Another case is the block diagonal form for the covariance which may be used to model a non-stationary environment.

Unfortunately, it turns out that for these covariance structures, with the signal bearing and waveform known, it becomes intractable to use the GLR procedure to obtain a test statistic.

Consequently, we approach signal detection from the viewpoint of the general theory of hypotheses testing. We model the signal strength μ as deterministic-unknown. This along with the unknown covariance matrix become the parameters describing the distribution of the observed data vectors. The problem of signal detection becomes one of choosing between two disjoint parameter sets based on the observations. Thus we have the following hypothesis testing problem:

Given

$$X_{NxL} \sim \mathcal{N}(\mu a e_1^H, R \otimes I) \tag{2}$$

where the columns of X are independent data vectors each normally distributed with covariance R as in (1), a is the signal vector (known), possibly present only in the first column and μ is its strength (unknown).

Test

$\mathrm{H0} : \mu = 0$ versus $\mathrm{H1} : \mu \neq 0$

Note that the covariance is a nuisance parameter which should not affect the decision statistic. This motivates us to reduce the problem as follows. Transformations on the data that induce transformations on the parameters to which the parameter sets are invariant leave the decision problem unchanged. Therefore, the decision statistic should also be invariant to all such transformations. More concretely, this can be formulated as follows:

Let X be the data characterized by the probability distribution P_θ, $\theta \in \Omega$ and let g be a $1:1$ onto transformation on the sample space such that gX is distributed as $P_{\theta'}$, $\theta' \in \Omega$. This transformation thereby induces a transformation $\bar{g}$ on the parameter space. It is shown in [4] that the set of all transformations g, such that the corresponding induced transformation $\bar{g}$ is a $1:1$ map of Ω onto itself, form a group.

The decision problem $\mathrm{H0} : \theta \in \Omega_0$ vs $\mathrm{H1} : \theta \in \Omega_1$
is invariant to the group of transformations, G, if $\bar{g}\Omega_i = \Omega_i$ $i = 0, 1$ for all $g \in G$. In that case we require the decision statistic to be invariant to all transformations in G.

This principle of invariance (Lehmann[4]) greatly reduces the class of detectors to be considered and frequently, it may become possible to find a uniformly most powerful test within this smaller invariant class (UMPI), even though no general UMP test may exist. Often, the GLR procedure leads to such a test. In our case, since the GLR is unavailable, we proceed by deriving the group of transformations that leave the problem invariant. From this, we obtain the maximal invariant, which is the algebraic basis for the largest set of independent functions of the data that are invariant to the transformations. These functions separate the sample space into orbits or invariant subsets. Thus, $M(X)$ is a maximal invariant iff

$$M(X) = M(g(X)), \forall g \in G \tag{3}$$

$$M(X_1) = M(X_2) \Rightarrow X_1 = g(X_2) \text{ for some } g \in G$$

It is shown in [4] that all invariant test statistics are functions of the maximal invariant, whose distribution depends on a reduced parameter set (this may eliminate the nuisance parameters from the problem, which is a very desirable feature). The maximal invariant turns out to be a small set and it is feasible to come up with a reasonable test statistic.

A Maximal Invariant Framework

To begin, consider as a special case of (1) the following structure

$$R = \begin{pmatrix} R_\psi & \mathbf{0} \\ \mathbf{0} & \sigma^2 I_{(N-d)L} \end{pmatrix} \tag{4}$$

This is completely equivalent to equation (1) with $R_\psi = B + \sigma^2 I_d$, since it can be obtained by a known linear transformation on the data. Again, for the same reason, we can assume the following form for the signal vector, a, and partition the data matrix accordingly:

$$a = \begin{bmatrix} a_1 \\ \mathbf{0} \\ a_2 \\ \mathbf{0} \end{bmatrix}, \quad X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \\ x_{41} & x_{42} \end{bmatrix} \tag{5}$$

where x_{11} is 1×1, x_{12} is $1 \times (L-1)$, x_{21} is $(d-1) \times 1$, x_{31} is 1×1 and x_{41} is $(N-d) \times 1$

We can represent this matrix as a length-NL vector

$$x = \begin{bmatrix} x_{11} \\ x_{21} \\ vec\begin{pmatrix} x_{12} \\ x_{22} \end{pmatrix} \\ x_{31} \\ x_{41} \\ vec\begin{pmatrix} x_{12} \\ x_{22} \end{pmatrix} \end{bmatrix} \tag{6}$$

where $vec(A) = [A_1^H \; A_2^H \; \ldots \; A_N^H]^H$ for $A = [A_1 \vdots A_2 \vdots \cdots \vdots A_N]$

This is distributed as a Gaussian vector with mean μa and covariance $diag(R_\psi \otimes I_L \; \sigma^2 I_{(N-d)L})$ where $\otimes$ denotes the Kronecker product.

This decision problem is invariant to all transformations which preserve the Gaussian nature of the distribution, the mean vector to a scale factor and the structure of the covariance matrix. The largest group of such linear transformations are given by $T(x) = Gx$ where

$$G = \alpha \begin{pmatrix} \begin{pmatrix} 1 & \beta^H \\ 0 & \Gamma \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & U_1 \end{pmatrix} & \mathbf{0} \\ \mathbf{0} & \begin{pmatrix} 1 & 0 \\ 0 & U_2 \end{pmatrix} \end{pmatrix} \tag{7}$$

where $U_1(L-1)$ and $U_2((N-d)L-1)$ are unitary matrices, β^H is $(1 \times L-1)$ and Γ is $(N-1 \times L-1)$.

We show in [7] that the maximal invariant to this group of transformations is given by

$$\begin{aligned} m_1 &= \frac{\| x_{11} - x_{12} x_{22}^H (x_{22} x_{22}^H)^{-1} x_{21} \|^2}{x_{12}(I - x_{22}^H (x_{22} x_{22}^H)^{-1} x_{22}) x_{12}^H} \\ m_2 &= x_{21}^H (x_{22} x_{22}^H)^{-1} x_{21} \end{aligned}$$

$$m_3 = \frac{\| x_{31} \|^2}{\| x_{32} \|^2 + \| x_{41} \|^2 + \| x_{42} \|_F^2}$$

$$m_4 = \frac{x_{11} - x_{12} x_{22}^H (x_{22} x_{22}^H)^{-1} x_{21}}{x_{31}} \qquad (8)$$

A corresponding maximal invariant in the parameter set is given by

$$\theta_1 = |\mu|^2 |a_1|^2 (R_\psi)_{11}^{-1}$$

$$\theta_2 = \sigma^2 (R_\psi)_{11}^{-1} \qquad (9)$$

Thus we have greatly reduced the dimensionality of the problem. We obtain the density function for the maximal invariant [7] which is now parameterized by θ_1 and θ_2 above. We show there that no UMP test exists for this problem. Further, since θ_2 is a free parameter even under **H**0, the distribution function of the maximal invariant is not completely specified thereunder and hence an invariant decision statistic will, in general, not have the CFAR property. Approximate CFARness is all one can hope for.

For the unstructured case, the maximal invariant reduces even further, to m_1 and m_2 and the corresponding parameter set to a single parameter θ_1 (which is the SNR). The distribution function under **H**0 only depends on the dimension of the data set, and so in this case, any invariant decision rule will be CFAR. Again, in this case no UMP test exists. However, in many applications the performance is critical only for low SNR and a locally most powerful invariant test(LMPI) in the limit of zero SNR is of interest. Since, the parameter space is one dimensional, it becomes feasible to obtain the LMPI test statistic following the theory in [5]. The LMPI decision rule in the limit of θ_0 is given by:

$$\frac{\frac{\delta f_\theta(x)}{\delta\theta} \Big|_{\theta=\theta_0}}{f_{\theta_0}} \underset{H_0}{\overset{H_1}{\gtrless}} \tau \qquad (10)$$

where f is the density function of x with parameter θ.

In [7] we derive the following density function for m_1 and m_2:

$$f(m_1, m_2) = k_1 \frac{(1+m_2)^{-N} m_2^{N-2}}{(1+m_1+m_2)^{L-N}} e^{-\frac{\theta_1}{1+m_1+m_2}} \sum_{k=0}^{L-N} k_2 \left(\frac{\theta_1 m_1}{(1+m_2)(1+m_1+m_2)}\right)^k \qquad (11)$$

where $k_1 = \frac{L-1!}{N-2!L-N-1!}$ and $k_2 = \frac{L-N!}{L-N-k!k!}$

Applying the rule in (10), we obtain the following LMPI test:

$$\frac{(L-N)t_K - 1}{(1+m_2)(t_K+1)} \underset{H_0}{\overset{H_1}{\gtrless}} \tau \qquad (12)$$

where $t_K = m_1/(1+m_2)$ is the Kelly statistic.

The pfa is closed form [7], and the detection probability is calculated numerically as a finite sum of simple integrals. Preliminary comparisons with the Kelly statistic indicate a slightly better performance at very low SNR (a gain of 0.1 dB for $N = 4, L = 9, pfa = .1$ at -5dB SNR)

at the expense of a degradation in the higher SNR region (0.3dB loss at 10dB SNR). Further simulations are shown in [7].

Finally, we note that m_1 is exactly the AMF statistic and the Kelly statistic is $m_1/(1+m_2)$. Thus these two form an equivalent basis set to m_1 and m_2. This implies that they form an algebraic basis for all invariant detectors and in searching for viable detectors, it is sufficient to look at compositions of them. It is not necessary to explore alternative ways of projecting down the initial raw high-dimensional data.

Detectors for subspace covariance structures

We consider above the case when the matrix is unstructured, as per Kelly and Reed. We saw that the Kelly and Reed tests are both equally valid since they are basis functions for all invariant detectors. We also found the LMPI test. We extended our work to the subspace structured covariance case [7]. This problem is important for enhanced detection because exploitation of covariance structure allows one to converge to reliable detection with fewer snapshots. ROC curves are shown in [7], which demonstrate that the invariant tests outperform previous ad hoc solutions.

Conclusion and Comments

Detection in an array environment involves projecting down the multivariate data to a scalar statistic. Since any reasonable statistic must satisfy the invariance criterion, the maximal invariant set specifies all the functions one need consider in devising the detector. Since this set is often small, it is feasible to do analysis and search for a detector with some optimality property with the confidence that the search is over the whole class of reasonable detectors. This framework provides an alternate route to conventional methods like the GLRT for arriving at decision rules and further enables the study of their optimality properties. Thus, for the unstructured covariance case studied by Kelly and others, we show that the Kelly and the AMF statistics form a maximal invariant set. Further, we show that a UMPI does not exist and obtain a LMPI test around 0 SNR. The Kelly detector is seen to perform nearly as well which is a good endorsement for its use. For the structured covariance case where the GLRT breaks down, we again obtain a small invariant set whose statistics can be analyzed. For this case, the UMP test does not exist.

We have derived extensions [7] of our test for cases when there are various unknown parameters in the target signal waveform and/or bearing. This has relevence in applications involving range migration such as occurs with accelerating targets. We are in the process, under a new AFOSR grant of (i) extending our work to testing for spatial stationarity as arises in faulty sensor and/or near field source detection, and (ii) applying our tests to real data.

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- "Invariant tests for Spatial Stationarity using Covariance Structure", S. Bose, A. Steinhardt, accepted for the 1993 ICASSP conference.

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